A Discussion of the light curve of the RV Tauri variable AC Herculis

by

E. A. KREIKEN

(Department of Astronomy)

Abstract: Payne - Gaposhkin, Brenton and Gaposhkin [1] have suggested that the complicated variations in the light curves of the RV Tauri variables may result from overtone pulsation. The hypothetical fundamental period interacts with the «essential» period thus causing the «classical» period. The ratios of the fundamental period to the other periods are equal to non integer values. In the autocorrelogram of R Scuti this fundamental period did not show up. There were two periods with a ratio 2:1. The first corresponded to the «classical» period and might be caused by variations of
the autoregressive type in the original light curve. The second corresponded
to the «essential» period and arose from harmonic terms in the original
curve. For no other RV Tauri variables is a light curve available which
meets the prerequisites of extended continuity and homogeneity which are
essential for the application of the autocorrelation method.

The question is considered whether the condition of continuity can be
dropped or not, many extended light curves being available. As an example
the light curve of AC Herculis is considered. In the correlogram and the
corresponding Power spectrum two periodicities are present. The periods are
in the ratio 2:1. The first corresponds to the «classical» period and may
be caused by autoregressive variations. The second corresponds to the
«essential» period and may be due to harmonic variations in the original
light curve. No indication is found for a fundamental period. The results can
only be accepted with certain reserve.

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1. In their extensive study of the RV Tauri variables, 
Payne - Gaposchkin, Brenton and Gaposchkin [1] tentatively
suggest that the observed irregularities in the light curves of
the RV Tauri stars are the result of their undergoing an overtone
pulsation. An hypothetical Cepheid like fundamental is
introduced, while it is supposed that for most of the RV Tauri
stars the amplitude of this fundamental is too small to effect
appreciably the light curves and the velocity curves. From the
relationship of the velocity curve and the light curve these
same authors conclude that when the RV Tauri stars are to be
considered as pulsating stars, the period of pulsating must be
half the «classical» period. Here the classical period is the in-
terval between successive similar minima. The period of pulsation
which is the «essential» period represents the first overtone of
the fundamental.

This suggestion is based on the results of the theoretical
work of Kuyper [2], Schwarzschild [3] a. o., from which it has
appeared that the ratio of the fundamental period and its first
overtone depends on the ratio of the specific heats. If the ratio
of the specific heats of the stellar material is between 1.43 and
1.67 the ratio of the periods is between 0.554 and 0.738.
Generally this latter ratio is a non integer.

In a recent paper [4] the present author has tried to ascertain
whether the simultaneous existence of several periodicities in
the atmosphere of RV Tauri stars could be confirmed by an
analysis of the observed light curve.
In this analysis the method used was the one introduced by Ashbrook Duncombe and van Woerkom [5] in their study of the light curve of \( \mu \) Cephei. To the light curve observed for this variable they first apply the autocorrelation method developed by Kendall [6] and thus obtain the correlogram. This correlogram is converted into the corresponding power spectrum. When the author applied this method to the light curve of the RV Tauri variable R Scuti an autocorrelation curve and a power spectrum were obtained in which two periods were clearly represented with a length of approximately 140 and 70 days. The 140 day period appeared to be strongly damped in the correlogram while as far as could be judged from the correlogram the 70 day period was undamped. The 70 day period, which obviously represents the “essential” period seemed to correspond to a harmonic term in the original light curve or a set of harmonic terms. The 140 day period represents the 146 day classical period and may not be a period in a strict sense. It may correspond to variations in the original light curve which are all of the same autoregressive type viz. \( U_{t+2} = -a U_{t+1} - b U_t - \varepsilon_{t+2} \). In the power spectrum also only periods of 140 and 70 day respectively showed up and no definite evidence was obtained for the occurrence of an additional “fundamental” period, and so it was concluded that it either is non-existent or has an amplitude which is too small to show up either in the correlogram or the power spectrum. The two periodicities which are present appeared to be exactly in the ratio 2:1. This conclusion based on the analysis of only one RV Tauri variable cannot as yet be considered as being definite. The question must be raised whether it can be extended to the other variables of this class as well. Obviously this question can only be settled by the analysis of additional light curves. But when we attempt to do this we are met by some difficulties.

2. The prerequisites of the autocorrelation method are extent continuity and reasonable homogeneity of the observed light curve. With the RV Tauri variables these specifications are completely met only by the light curve of R Scuti. In all other curves seasonal gaps occur. It is evident that especially the conditions of continuity drastically cuts down
the usefulness of the autocorrelation method in all astronomical work.

With the long period variables conditions of visibility and climatic conditions during part of the year will make observations impossible or at least very difficult. As a result gaps will occur in the observed light curve while the spacing of these gaps is periodical. Similarly in the observed light curves of the short period variables a series of interruption is bound to occur spaced at intervals of one day. Therefore in this case also there is a periodicity in the occurrence of the interruptions. Consequently of the three afore mentioned prerequisites the one of continuity seems the most difficult to meet. The conditions of extent and homogeneity are satisfied in many instances. It therefore seems worth while to consider the question whether the condition of continuity is entirely indispensable. Normally when a continuous series of observations is available we compute the numerical value of the autocorrelation coefficients \( r_k \) from the convenient approximation used by Ashbrook, Duncombe and, van Woerkom (1. c).

\[
r_k = \left( \sum_i^N i U_i U_{i+k} \right) \left( \sum_i^N i U_i^2 - K U^2 \right) \quad \cdots (1)
\]

which is adequate for \( N \ll k \). The correlogram is obtained by plotting the resulting values of \( r_k \) against the corresponding values \( k \).

With the autocorrelation method equidistant observations must be used which are obtained by dividing the observed light curve in a large number of equidistant steps. When in the observed light curve a number of gaps occur it is still possible to divide the total time interval in equidistant steps, but now a number of these steps coincide with the gaps and for those no observed values \( U_i \) are available. Now let \( N_1, N_2, \ldots, N_{n+1} \) be the number of steps within the successive intervals covered by observations and \( G_1, G_2, \ldots, G_n \) the number of steps within the successive gaps. Both with the seasonal interruptions and the diurnal ones the numbers \( N_1 \ldots N_n \) and \( G_1 - G_{n-1} \) will be more or less equal inter se. The question which must be considered is, whether we still obtain reliable results if when computing the correlation [1] we periodically drop a certain number
of terms $U_i$. Provided that the intervals $N$ are of sufficient extent and the intervals $G$ are short, there do not in general seem to be many a priori objections. Obviously the periodicities to be expected in the observed light curve and in the correlogram should be of such a length, that the whole length of an oscillation is uniformly covered by the successive intervals $N$. The real danger is that we can never be entirely sure about this point. On the other hand, if relatively to the intervals $N$, the various periodicities are small and the ratio of the periodicities and the lengths of the intervals $N$ and $G$ can not be expressed as the ratios of small integer numbers we may be reasonably sure that by summation of the products derived from an intermittent curve we can obtain results which are not far inferior to those obtained from a continuous curve. The analytical expression for $r_k$ derived from a light curve containing a number of gaps would be:

$$r_k = \left\{ \frac{N_{1+G_1+N_2-K}}{N_{1+G_1+1}} \times \frac{N_{1+\ldots+N_{n+1}+G_1\ldots+G_n-k}}{N_{1+\ldots+N_n+G_1\ldots+G_n+1}} \right\} \times \left\{ \sum_{i}^{N_{1-k}} \sum_{i+k}^{N_{1+k}} U_i U_i^2 \ldots \sum_{i}^{N_{1+k}} U_i^2 \sum_{i}^{N_{1+k}} U_i \right\}^{-1/2}$$

or written in a more compact form:

$$r_k = \left\{ \sum_{j=0}^{n} \frac{A_j-k}{L_j} \right\} \left\{ \sum_{j=0}^{n} \frac{A_j-k}{L_j} \times \sum_{j=0}^{n} \frac{A_j}{L_j} \right\}^{-1/2} \ldots (2)$$

where $L_j = 1 + \sum_{j=0}^{j}(N_j + G_j)$ and $A_j = \sum_{j=0}^{j}(N_{j+1} + G_j)$ with $N_0 = G_0 = 0$.

When the intervals $N$ are of sufficient extent we could still use the approximation [1] so that instead of [2] we can write:

$$r_k = \left\{ \sum_{j=0}^{n} \frac{A_j-k}{L_j} \right\} \left\{ \sum_{j=0}^{n} \frac{A_j}{L_j} \right\}^{-1} \ldots (3)$$

where $L_j$ and $A_j$ have the same meaning as before.
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The relation (2) and its substitute (3) look much more formidable than the simple relation (1). In the actual computations the difference is less than would be expected. It simply means that the computations are carried out in the usual way, but that at given intervals a certain number of terms $U_i$ and $U_{i+k}$ has to be dropped.

We can also so say that the calculations are first carried out for the separate intervals while next the results for all intervals are summed. By a suitable arrangement of the observed values $U_i$, these various operations can be carried out without any great difficulties. However, the values $r_k$ derived from (3) will be less reliable than those derived from a continuous curve.

For evaluating the mean error affecting the values $r_k$ derived from a continuous set of observations Ashbrook, Duncombe and van Woerkom use a relation derived by Bartlett [7], viz.

$$\sigma_k^2 = N^{-1} (1 + 2r_1^2 + 2r_2^2 + \ldots + 2r_k^2) \ldots (4)$$

Here $N$ is the number of steps used in the computation of $r_k$. With a continuous series of observations $N$ decreases by unity with each successive value of $k$. That is to say that there is a slow but steady increase in the mean error with increasing value of $k$.

If the relation (4) is assumed to remain valid for an intermittent curve (and this may be true provided that the conditions are met stated in the beginning of this section), it is at once evident that the increase of $|\sigma|$ is considerably larger than in the previous case. To an increase of $k$ by one there now corresponds a decrease of $n$ in $N$ and for $r_k$ the decrease is $n \cdot k$.

3. When the duration of the gaps is short the intervals $G_k$ are short and especially with the higher values of $k$ it may occur that $k \gg G_j$. In these cases some additional terms must be included in the relations (2) and (3). No further explanation is needed to see that now instead of (2) we have:

$$r_k = \left\{ \sum_j n \left[ \frac{A_j - k}{L_j} \frac{A_j}{U_i U_{i+k} + \sum_i U_i U_{i+k}} \right] \right\} \left\{ \sum_j \frac{n}{L_j} \left( \frac{A_j - k}{U_i^2 + \sum U_i^2} \frac{A_j}{A_j + G_j + 1 - k} \right) \right\}^{-1/2} \ldots (5)$$

$$\left[ \sum_j \left( \frac{A_j - k}{L_j} \frac{A_j}{U_i^2 + \sum U_i^2} \right) \right]^{-1/2} \ldots (5)$$
while instead of (3) we have:

\[ r_k = \left\{ \sum_{j=0}^{n} \left[ \sum_{i} U_{j} U_{i+k} + \sum_{i} U_{i+k} \right] \right\} \left[ \sum_{j} \left( \sum_{i} U_{j}^2 \right)^2 \right] \left( \sum_{G \neq k} k \right)^{-2} \cdots (6) \]

Here again the calculations are far less complicated than suggested by this analytical form. They merely state that the normal calculations are to be carried out for each interval but an additional number of terms has to be included which result from the overlapping of the values \( U_{i+k} \) with the values \( U_i \) in the next interval. As before the difficulty is purely formal and can be largely reduced by a suitable tabulation of the observed values \( U_i \). The work which is involved in the calculations of the values \( r_k \) is considerable, but not much in excess of that which is needed to find the values \( r_k \) from a continuous series of observations. However, just as in the case considered in section (2) there is a danger that the length of the periodicities in the original light curve and in the correlogram are of the same order of magnitude as the periodicities in the occurrence of the gaps or that at least the ratios of the various periodicities can be expressed by simple integer values. The result would be that the correlogram is seriously distorted. If we allow \( k \) to increase further at a given moment we will have \( k \geq A_j \) and as a consequence the values \( U_i \) are in the one observed interval and the values \( U_{i+k} \) in the next one. It would not be difficult to draw up an additional analytical expression covering this case. It would be hardly justified to do so. Such an expression would only indicate that for all possible combinations of the observed values we must compute:

a) The sum of the products \( U_i \cdot U_{i+k} \)

b) The sum of the squares of all values \( U_i^2 \) while from this sum we must subtract as many times the mean value \( U_i^{-2} \) as we had to drop terms \( U_i \) and finally:

c) The ratio of these two sums.
Within this additional range of \( k \), the number of values which can be included in the calculations at first increases but later on sharply decreases again. Consequently over the whole range of values \( k \) the number \( N \) varies in a rather complicated way. But even with the higher values of \( k \) the objection remains that for the steps which coincide with the gaps no observed values \( U_i \) are available. So just as in the previous cases periodically a number of terms \( U_i \) have to be omitted, So the danger remains that this periodicity may distort the normal periodicities to be expected in the correlogram.

Summarising we find that is may be worth while to compute the correlogram even when only an intermittent light curve given, but that caution is needed when interpreting the periodicities in the correlogram.

4. From the RV Tauri variables discussed by Payne-Gaposchkin, Brenton and Gaposchkin [1] AC Herculis is the one which has been most extensively observed.

The aforementioned authors give the light curve between J. D. 24 23 000 and J. D. 24 30 580. The period as determined from two successive minima is 75 d. 24 and this therefore represents the «classical» period.

From examination of this light curve it is evident that it is a typical example of a light curve affected by seasonal circumstances. Notwithstanding the fact that the variable has been extensively observed, the light curve contains a series of gaps. These gaps are fairly regularly spaced at one year intervals. For the application of the autocorrelation method aequidistant observations are required. With a view to the observed period the interval between J. D. 24 23 000 and J. D. 24 30 580 was divided in aequidistant steps of five days each. Obviously the gaps had to be included. Next from the observed curve given by Payne-Gaposchkin at all. the values \( U_i \) corresponding to each step were read. Here \( U_i = m_i - \bar{m} \) and \( \Sigma U_i = 0 \). Both from an inspection of the light curve and from the values \( U_i \) it appears that no secular trends are present. I have refrained from tabulating the individual values \( U_i \) as these can directly been read from the observed curve.
Obviously values $U_i$ are available only for the intervals covered by observations. These intervals have been indicated by $N_j$, within the intervals $G_j$ we do not have any values $U_i$. As stated before when splitting up the total light curve in equidistant steps, the intervals $G_j$ were included. If the step coinciding with J. D. 2422000 is taken to be zero, the intervals $N_j$ and $G_j$ expressed in equidistant steps are as indicated in Table I. Excluding the last value $N_j$ it appears from the table that the mean lengths of the intervals $N_j$, $G_j$ and $N_j + G_j$ are 53, 20 and 73 respectively. As was to be expected the latter number of steps corresponds to a time interval of one year. It is also evident that the scatter of the individual values $N_j$, $G_j$ and $(N_j + G_j)$ round these means is fairly large. The sum of the numbers $N_j$ indicate the maximum number of values $U_i$ which can be used in the calculations. This sum is equal to 1046. This is a satisfactory number of values $U_i$ but it can only be used when computing the numerical value of $r_k$ for $k = 0$ (which is equal to one by definition). As explained in sections (2) and (3), with increasing values $k$, the number of values $N$ which can be used at first decreases rapidly, next remain constant or almost constant until an increase sets in.

After a certain maximum is attained a new rapid decrease of the available number of values $U_i$ sets in.

The total numbers of values $U_i$ which can be used in the calculations has been indicated by $N(k)$. The numerical values of $N(k)$ have been tabulated in Table II. They are important for evaluating the mean error affecting the autocorrelation coefficients $r_k$ which are also given in Table II. These autocorrelation coefficients have been computed in the way indicated in the preceding sections. From Table II we obtain the correlogram by plotting the values $r_k$ against the corresponding values of $k$. The results appear in Figure I. Before discussing the implications of the correlogram a few words must be be said about the mean error of the values $r_k$ on which the correlogram is based. According to Bartlett [7] the variance of $r_k$ is given by the equation (4) provided that there are no sampling errors. As sampling errors are present, the mean error should be larger than the values $\sigma$ computed from the equation (4). The values $\sigma$ as computed from (4) are also indicated in Figure I. For this it had to be assumed that the relation given by Bartlett
Table I. The intervals \( N_j \) and \( G_j \) expressed in steps. Numbers of steps within the intervals (sections 2 and 4).

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<th>( N_j )</th>
<th>( \Sigma i )</th>
<th>( G_j )</th>
<th>( \Sigma i )</th>
<th>( j )</th>
<th>( N_j )</th>
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Table II. Tabulation of the values \( r_K \) and \( N(K) \)

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could also be applied in the present case. The observed
numbers N(k) of values U_i being large I have considered the pos-
sibility of evaluating the mean error affecting the values r_k in
a purely empirical way.

By combining the values U_i of the successive intervals of
Table I, we can compute values r_k for each interval separately.

In that way 19 different sets of values r_k are obtained and by
intracomparsion we obtain a second set of values |σ|. In
figure I this second set of values is indicated by the curve
|σ| obs. The results are not very encouraging. For the whole
range of the values r_k we have |σ| obs < |σ| th. With the
smaller values of k this difference is small, but it becomes more
and more pronounced as k increases. With the partial curves
r_k only short ranges of values U_i are used.

Within these small ranges the different periodicities may not
have sufficient opportunity to get out of tune. Consequently the
small values |σ| obs may be misleading. In the following only
the values |σ| as computed from theory are used and even
these values indicate certain minimum values. The actual mean
errors may be even larger than these minima.

When considering the autocorrelation curve in Figure I, two
periodicities are at once apparent. The lengths of the two
periods are 15×5 and 7.5×5 days respectively. Apparently the
75 day period corresponds to the "classical" period and the 37.5
day period to the «essential» period of AC Herculis. Apart from the maximum at $k = 0$, which is equal to 1 by definition, the intensity of the successive maxima seems to remain constant for both periodicities. In this respect the correlogram of AC Herculis is different from that obtained for R Scuti (4). With R Scuti only the «essential» period seemed to be undamped, while the classical period was strongly damped and might correspond to variations of the autoregressive type in the original light curve. I do not think however that in the present case our conclusions can be very definite. With the higher values of $k$ the correlogram in Figure 1 is affected by large uncertainties and here especially the periodic character of the successive gaps in the observed light curve might affect its shape. The only conclusion which seems pretty certain is that just as with R Scuti, the two periodicities which occur in the light curve of AC Herculis are in the ratio $2 : 1$.

5. By applying the relation:

$$\pi(f) = \frac{2}{N} \sum_{0}^{N} r_k \cos 2 \pi f k$$

the correlogram is converted into the corresponding power spectrum. Here $f$ is the reciprocal of the trial period. In their study of the light curve of $\mu$ Cephei Ashbrook, Dun ombe and van Woerkom [5] sharply cut down the number of terms $r_k$ which they use in the computation of the numerical values of $\pi(f)$. The reason is that they want to avoid the inclusion of terms which are statistically insignificant. The present author has shown [8] that there is some danger in using too small a number of values $r_k$. When the number of terms $r_k$ is only slightly larger than the number of terms covering one complete period of the correlogram, some systematic effects may be introduced which distort the power spectrum. The result may be that the maximum in the power spectrum corresponding to the principal period is broadened. At the same time some spurious maxima and minima may be thrown up. On the other hand when including the higher values $r_k$ we do not only consider less reliable values of the autocorrelation coefficient, but there will also be an increasing danger that the periodical gaps in the observed light curve affect the shape of the power spectrum. It is therefore
rather difficult to fix the limit of the values $r_k$ in a wholly unobjectionable way. Consequently I have preferred to compute the power spectrum using different limiting values of $r_k$. The result is that we obtain several power spectra and by comparing these different curves it will become apparent how the shape of the power spectrum is affected by the choice of the limit. In Table III I have tabulated the curves $\pi(f)$ which are obtained if for the limit of $r_k$ we adopt the values $k = 20, 30, 40$ and 60 respectively. The first column of the table contains the reciprocal of the trial period. A graphical representation of

![Graph](image)

Fig. II. Power Spectrum of AC Hercules corresponding to the correlogram in Figure I.

some of the curves is given in Figure II. From an inspection of the numbers in Table III and of the curves in Figure II it appears that in the power spectrum corresponding to $k = 20$ only two clear cut maxima occur. The approximate position of these maxima is at $1/f = 15$ and $1/f = 7.5$. They correspond to the periodicities which already occurred in the correlogram represented in Figure I. No further maxima are present. The maximum at $1/f = 7.5$ is sharp and narrow and may therefore
correspond to a harmonic term in the original light curve. The maximum at $1/f = 15$ has a slightly greater intensity but appears to be somewhat broadened. This might be due to the fact that the 75 day period is not a period in a strict sense, but corresponds to a series of variations of the autoregressive type. In this case however, the evidence is hardly convincing, because the broadening may also be due to the fact that when computing this first power spectrum a limit of $k$ was used, which is only slightly larger than the 75 day period.

When the power spectrum corresponding to $k = 30$ is considered it appears that the maximum around $1/f = 7.5$ starts to disintegrate. A faint secondary maximum appears at $1/f = 10$ exactly at the place where in the curve corresponding to $k = 20$ a minimum is found. With the curve $k = 30$, we are using for the first time values $r_k$ which are derived from observed values $U_i$ and $U_{i+k}$ corresponding to two successive intervals $N_j$ and $N_{j+1}$ which are separated by a gap $A_j$. The mean length of $G_j$ being 20 (see section 4), this secondary maximum may not be real. It is still apparent in the power spectrum corresponding to $k = 40$, but now the two principal maxima are of almost equal intensity. With $k = 60$ the disintegration of the power spectrum is almost complete. The secondary maximum at $k = 10$ has dissappeared, but at the same time the original maximum at $1/f = 7.5$ has split up into two parts. The maximum at $1/f = 15$ is flattened and the power spectrum contains an additional maximum (at $1/f = 24$) which is neither shown in Table III nor in Figure II. It was suppressed because it is evident that the power spectrum corresponding to $k = 60$ is valueless.

The two only maxima therefore which seem to be well established are those corresponding to the classical and essential period respectively. We do not obtain evidence for the presence of a third period, corresponding to the fundamental period.

6. In their original paper Payne-Gaposchkin et al. state that they expect the amplitude of the hypothetical fundamental period to be small. Therefore the failure of such a fundamental period to show up in the correlogram and power spectrum of both R Scuti and AC Herculis does not prove that a funda-
mental period is not present. A more serious objection is that the periodicities which clearly show up in the correlograms and power spectra of these two RV Tauri variables are exactly in the ratio 2:1. With R Scuti we concluded that one of the periods (the longer one) is not a period in a strict sense. This may also be the case with AC Herculis, but with this latter

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star the evidence is in no way conclusive. For the present the question whether the longer period is a period in a strict sense or not remains debatable.

It seems dubious whether this question could be settled by an analysis of RV Tauri stars in the way described in the present paper. Not only does such an analysis require a considerable amount of calculations, but also the final results are not established with such a degree of reliability as must be desired.
On the other hand by now we can accept as a well established fact that only periodicities occur which are in the ratio 2:1. It may be difficult to reconcile this observational fact with a theory based on the simultaneous existence of a fundamental and its overtones in the atmosphere of a variable of which the ratios cannot as a matter of principle be represented by simple integer numbers.

For the present I would like to repeat the tentative conclusions derived as the result of an analysis of the light curve R Scuti. These conclusions were:

a) The atmospheres of the RV Tauri stars are affected by two periodicities which are in the ratio 2:1.

b) The shorter period, which corresponds to the essential period and to the period in the radial velocity curve, is the result of an harmonic term or the sum of harmonic terms in the original light curve.

c) The longer period which is exactly twice as long as the previous one and which corresponds to the classical period may not be a period in a strict sense. It may result from a series of autoregressive variations in the original light curve. With AC Herculis this latter point is far less evident than with R Scuti.

Literature


(Manuscript received on May 1957)