Pion-Gauge Conditions for $N\phi \rightarrow N^*\pi$ Reaction

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**Pion-Gauge Conditions for $Np \to N^*\pi$ Reaction**

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General self-consistency conditions are derived for $N^* j = 1/2$ production amplitudes and coupling constants by the methods of pure S-matrix theory. The similarity to electroproduction is discussed.

We consider

$$N(k_i) + g(k_2) \to N^*(k_3) + \pi(k_4)$$  \hspace{1cm} (1)

reaction (s-channel) with masses $k_1^2 = m_1^2$, $k_2^2 = \lambda^2$, $k_3^2 = m_3^2$, and $k_4^2 = \mu^2$. $N$ and $N^*$ are both spin $1/2$, isospin $1/2$ nucleon resonances. The spin basis for the reaction (1) is well known from electroproduction, only a slight modification is needed to take care of the unequal masses of the nucleons:

$$V_i = u(k_3) \gamma_5 v_{i\mu}^\dagger u(k_1) \varepsilon_{\mu}(k_2)$$

where

- $v_1^{\dagger\mu} = \gamma_\mu k_2$
- $v_2^{\dagger\mu} = -\gamma_\mu$
- $v_3^{\dagger\mu} = 2 P_{\mu}$
- $v_4^{\dagger\mu} = 2 k_{4\mu}$

with $P = (k_3 + k_1)/2$. The isospin basis is same as with isovector electroproduction basis. The number of independent spin amplitudes is reduced to six by the condition $k_3 \cdot \varepsilon(k_2) = 0$.

In the limit $k_4 \to 0$, the undetermined part of the scattering amplitude will come from the contribution of the $N$ and $N^*$ in-
termediate states to Born terms. We give in Table I these terms, where we introduced the following form factors.

\[ g^{}(\mu^2) = g_{NN \pi} (m_1^2, m_1^2, \mu^2); \quad f'_{132}(\lambda^2) \equiv f'^{\nu}_{132NN \rho} (m_1^2, m_1^2, \lambda^2) \]

\[
g'(\mu^2) = g'_{N N \pi} (m_1^2, m_1^2, \mu^2); \quad f'_{132}(\lambda^2) \equiv f'^{\nu}_{132NN \rho} (m_1^2, m_1^2, \lambda^2) \]

\[
g''(\mu^2) = g''_{N N \pi} (m_1^2, m_1^2, \mu^2); \quad f''_{132}(\lambda^2) \equiv f''^{\nu}_{132NN \rho} (m_1^2, m_1^2, \lambda^2) \] (2)

All form factors have been assumed to be real and symmetric in the first two variables, \(m_1^2\) and \(m_3^2\). The \(N^*N\rho\) vertex is written as

\[
\bar{u}(k) \left[ \gamma^\mu f'_1(\lambda^2) + \frac{i}{2} \sigma_{\mu\nu} k^\nu f'_2(\lambda^2) \right] u(k_1) e^{ik_2} (k_2) \]

**TABLE I**

\[
B_1^+ = g^' \left( \frac{f_1 + 2m_1 f_2}{s-m_1^2} \pm \frac{f_1'' + 2m_1 f_2''}{u-m_3^2} \right) + [f_1' + (m_3 + m_1) f_2'] \]

\[ \quad + \left( \frac{g''}{s-m_3^2} \pm \frac{g}{u-m_1^2} \right) \]

\[
B_2^\pm = -g^1 \left( \frac{f_1}{s-m_1^2} \pm \frac{f_1''}{u-m_3^2} \right) - f'_1 \left( \frac{g''}{s-m_3^2} \pm \frac{g}{u-m_1^2} \right) \]

\[
B_3^\pm = -\frac{1}{2} \left[ g^1 \left( \frac{f_1}{s-m_1^2} \mp \frac{f_1''}{u-m_3^2} \right) \right] - \frac{1}{2} f'_1 \left( \frac{g''}{s-m_3^2} \mp \frac{g}{u-m_1^2} \right) \]

\[
B_5^\pm = g^1 \left( f_2 \mp f_2'' \right) + f'_2 \left( \frac{s-m_1^2}{s-m_3^2} \frac{u-m_3^2}{u-m_1^2} \right) \]

\[ \quad + (m_3-m_1) \left( \frac{g''}{s-m_3^2} \pm \frac{g}{u-m_1^2} \right) \]

\[
B_6^\pm = 2g^1 \left( \frac{f_2}{s-m_1^2} \pm \frac{f_2''}{u-m_1^2} \right) + 2 f'_2 \left( \frac{g''}{s-m_3^2} \pm \frac{g}{u-m_1^2} \right) \]

\[
B_8^\pm = g^1 \left( \frac{f_2}{s-m_1^2} \mp \frac{f_2''}{u-m_3^2} \right) + f'_2 \left( \frac{g''}{s-m_3^2} \mp \frac{g}{u-m_1^2} \right). \]
The normalization is same as in the references\textsuperscript{[1–2]}

We shall continue the total amplitude $M = B_i V_i$ to the unphysical point:

$$s = m_2^2, \quad u = m_1^2, \quad t = \lambda^2 \quad (k_4 \to 0) \quad (3)$$

For this purpose we separate the scalar $B_i$ amplitudes into Born terms coming from $N$ and $N^*$ intermediate states and the remainder part which is regular at the point (3).

$$M = (B_i^R + B_i^B) V_i \quad (4)$$

We also separate the basis functions $V_i$ into their finite and vanishing parts when $k_4 \to 0$, by the following identities:

$$V_1 = \gamma_2 + \bar{u}(k_5) \gamma_5 \gamma_\mu k_4 k_4 \gamma_\mu u(k_1) \varepsilon^\mu (k_2)$$

$$V_2 = \gamma_2 - (m_3 - m_1) \gamma_1$$

$$V_3 = 2 \bar{u}(k_5) \gamma_5 k_4 \gamma_\mu u(k_1) \varepsilon^\mu (k_2)$$

$$V_5 = -\gamma_1$$

$$V_6 = \frac{m_3 + m_1}{2} [\gamma_2 - (m_3 - m_1) \gamma_1] + \bar{u}(k_5) \gamma_5 P_\mu k_4 u(k_1) \varepsilon^\mu (k_2)$$

$$V_1 = \bar{u}(k_5) \gamma_5 k_4 \gamma_\mu k_2 u(k_1) \varepsilon^\mu (k_2)$$

Where

$$\gamma_1 = \bar{u}(k_5) \gamma_5 \gamma_\mu u(k_1) \varepsilon^\mu (k_2)$$

$$\gamma_2 = -i \bar{u}(k_5) \gamma_5 \sigma_{\mu\nu} u(k_1) \varepsilon^\mu (k_2) (k_j - k_i)$$

are the two spin amplitudes of the $NN^*\pi$ vertex with a pion spurion.

We evaluate the limit of Eq. (4) as $k_4 \to 0$, using (5). From the regular parts, only $B_i^R, B_2^R, B_5^R$ and $B_6^R$ survive. In the Born-terms we use the method developed in references\textsuperscript{[1–2]}: For the terms of the type $1/s = m_3^2$ we put first $k_4 = ak_3$ and then $a \to 0$; for the terms of the type $1/u = m_1^2$, we put $k_4 = -\beta k_1$ with $\beta \to 0$. We obtain by this method
\[ \pm \quad M_{k^4=0} = \{ B_1^+ + B_2^+ - \frac{m_3 + m_1}{2} B_6^- - \frac{g'(o)}{m_3 + m_1} (f_2 \pm f_2'') \]

\[ - f_2' \left( \frac{g''(o)}{m_3} \pm \frac{g(o)}{m_1} \right) \} y_2 \]

\[ - \{ B_5^+ + (m_3 - m_1) (B_2^+ - \frac{m_3 + m_1}{2} B_6^-) - \frac{g'(o)}{m_3 + m_1} \]

\[ \times (f_1 \mp f_2'') \} - \frac{1}{2} f_1' \left( \frac{g''(o)}{m_3} \mp \frac{g(o)}{m_1} \right) - f_2' (g''(o) \mp g(o)) \} y_1 \quad (7) \]

The pion gauge condition, \[\lim M = 0\] gives the following consistency conditions at the point (3):

\[ \pm \quad B_1^+ + B_2^+ - \frac{m_3 + m_1}{2} B_6^- = \frac{g'(o)}{m_3 + m_1} (f_1 \pm f_2'') \]

\[ + \frac{1}{2} f_2' \left( \frac{g''(o)}{m_3} \pm \frac{g(o)}{m_1} \right) \]

\[ \pm \quad B_5^+ + (m_3 - m_1) (B_2^+ - \frac{m_3 + m_1}{2} B_6^-) = \frac{g'(o)}{m_3 + m_1} (f_1 \mp f_2'') \]

\[ + \frac{1}{2} f_1' \left( \frac{g''(o)}{m_3} \mp \frac{g(o)}{m_1} \right) + f_2' (g''(o) \mp g(o)). \quad (8) \]

As \( B_1^- \), \( B_2^- \), \( B_5^+ \) and \( B_6^- \) amplitudes are antisymmetric under the interchange

\[ s \leftrightarrow u, \quad m_3 \leftrightarrow m_1 \]

they must be odd functions of the equally antisymmetric variable

\[ v = \frac{2}{m_3 + m_1} k_4 \cdot P = \frac{1}{2(m_3 + m_1)} (s - u - m_3^2 + m_1^2) \quad (10) \]

Hence in the limit \( k_4 \to 0 \), the constant terms in (8) must vanish. This implies the conditions:
\[
g'(o) \frac{1}{m_3+m_1} (f_2'-f_2''') + \frac{1}{2} f_2' \left( \frac{g''(o)}{m_3} - \frac{g(o)}{m_1} \right) = 0
\]  
(11)

\[
g'(o) \frac{1}{m_3+m_1} (f_1'-f_1''') + \frac{1}{2} f_1' \left( \frac{g''(o)}{m_3} - \frac{g(o)}{m_1} \right) + f_2' \frac{g''(o)}{g(o)} = 0
\]  
(12)

From Eq (11) we obtain

\[
g'(m_3^2, m_1^2, o) = \frac{g''(m_3^2, m_1^2, o)}{m_3} = \frac{g(m_1^2, m_1^2, o)}{m_1}
\]  
(13)

and with these conditions the Eq. (12) gives:

\[
f_2^{\gamma'} (m_3^2, m_1^2, \lambda^2) = 0
\]  
(14)

The conditions (13) are common to all reactions of the type \(NX \rightarrow N^*\pi\) with \(X = \pi, \gamma, \rho\) \([1-2]\). But the last condition on the magnetic isovector form factor of the vertex \(NN^*\rho\) is new.

The self consistency conditions (8) become, using (13) and (14)

\[
\begin{align*}
B_1^+ + B_2^+ & - \frac{m_3+m_1}{2} B_6^+ \bigg| \frac{g''(o)}{m_3} - \frac{g(o)}{m_1} \bigg| k_4 = 0 \\
B_5^+ & + (m_3-m_1) \left( B_2^+ - \frac{m_3+m_1}{2} B_6^+ \right) \bigg| \frac{g''(o)}{m_3} - \frac{g(o)}{m_1} \bigg| k_4 = 0
\end{align*}
\]  
(15)

In the equal mass case, we cannot directly take \(m_3 \rightarrow m\), limit of Eq. (15) because \(f_2^{\gamma'} (m_3^2, m_1^2, \lambda^2)\) is discontinuous. Since, \(f_2^{\gamma'} (m_1^2, m_2^2, \lambda^2) \neq 0\) in opposition to the condition (14) obtained as a consequence of the pion gauge condition and of the symmetry properties of the scattering amplitude under (9). We have to go back to the expressions of Born terms before \(k_4 \rightarrow 0\) limiting process. We have to take first \(m_3 = m_1\) and then to go to \(k_4 \rightarrow 0\) limit.
The procedure is same, we give in Table II the Born terms due to $N$ and $N^*$ intermediate states.

\begin{table}
\begin{align*}
B_1^\pm &= g \left( f_1 + 2m f_2 \right) \left( \frac{1}{s-m^2} \pm \frac{1}{u-m^2} \right) \\
B_2^\pm &= -g f_1 \left( \frac{1}{s-m^2} \pm \frac{1}{u-m^2} \right) \\
B_3^\pm &= -\frac{1}{2} g f_1 \left( \frac{1}{s-m^2} \mp \frac{1}{u-m^2} \right) \\
B_5^\pm &= g f_2 (1 \mp 1) \\
B_6^\pm &= 2g f_2 \left( \frac{1}{s-m^2} \mp \frac{1}{u-m^2} \right) \\
B_1^\mp &= g f_2 \left( \frac{1}{s-m^2} \mp \frac{1}{u-m^2} \right)
\end{align*}
\end{table}

The pion gauge condition gives in this equal mass case the following consistency conditions:

\begin{align*}
B_1^r + B_2^r - m B_5^r \bigg|_{k_i=0} &= \frac{1}{2m} g f_2 (1 \pm 1) \\
B_5^r \bigg|_{k_i=0} &= \frac{1}{2m} g f_1 (1 \mp 1)
\end{align*}

We observe that when $m_3 \neq m_1$, $B_1^r + B_2^r - m_3 + m_1$, $B_6^r = 0$ at the limit $k_4 = 0$. But when $m_3 = m_1$, the same
amplitude is not zero: \[ B_1^+ + B_2^+ = \frac{1}{m} g f_2. \] This is similar to \( A_{s^-} \) amplitude in electroproduction[2] where the discontinuity of \( f_1' \left(m_3^2, m_1^2, \lambda^2\right) \) plays a similar role.

**CONCLUSION**

We obtained, starting from the physical amplitude two sets of Adler type consistency conditions (15) and (16). We notice once more that, \( k_1 \rightarrow o \) and \( m_1 \rightarrow m_1 \) limits do not commute, due to the discontinuities of form factors.

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**REFERENCES**


**ÖZET**

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