REAL OPTIONS, INVESTMENT AND FINANCING DECISIONS, AND THE THEORY OF THE FIRM

Asım Gürsel ÇELİK*

I. INTRODUCTION

In the agency theory of the firm, Jensen and Mackling (1976) defines the corporations as legal fictions which serve as a nexus for a set of contracting relationships among individuals.1 The agency costs explanation of the theory of the firm helps us to understand better the nature of the relationships among owners, managers, debtholders, employees, suppliers, costumers and the regulatory power.

In another seminal paper, Black and Scholes (1973) provides a key to the valuation of contingent claims which, like options, have payoffs that are contingent on the future value of another asset. They not only enhance our understanding of financial investments but also show that if the firm's cash flow distribution is fixed, the option pricing analysis can be used to value other contingent claims such as the equity of a levered firm. In this context, the equity of a levered firm is a call option on the total value of the firm's assets with an exercise price equal to the face value of the debt and the expiration date equal to the maturity date of the debt.

More recently, financial economists have come to view investment opportunities as real options, exploiting an analogy with the theory of options in financial markets.2 These developments in corporate finance lead us to consider the firm as a nexus for a set of options contracts among individuals.

*Ph. D. student, Department of Finance, The University of Texas at Arlington, Box 19449 Business Bldg., Arlington, Texas 76019, U.S.A.

1 The antecedents of their work are in Coase (1937) and Alchian and Demsetz (1972).

The purpose of this paper is to investigate the possible effects of real options on firm's investment and financing decisions. The paper consists of four sections. Following introduction, section two discusses real options that embedded in investment decisions and provides a numerical example for valuing real options. Section three attempts to explain the effects of real options on firm's investment and financing decisions. This section also outlines a model for firm valuation in contingent claim analysis framework. Section four concludes.

II. REAL OPTIONS

Net Present Value (NPV) and other discounted cash flow (DCF) approaches to capital budgeting are incomplete in the sense that they cannot properly capture managerial flexibility. In practice, however, as new information arrives and uncertainty about market conditions and future cash flows is gradually resolved, management may have valuable flexibility to alter its operating strategy in order to capitalize on favorable future opportunities or mitigate losses. For example, management may be able to defer, expand, contract, abandon, or otherwise alter a project at different stages during its useful operating life.

Management's flexibility to adapt its future actions in response to altered future market conditions expands an investment opportunity's value by improving its upside potential while limiting downside losses relative to management's initial expectations under passive management.

An options approach to capital budgeting has the potential to conceptualize and even quantify the value of options from active management. This value is manifest as a collection of real options embedded in capital investment opportunities, having as an underlying asset the gross project value of expected operating cash flows. Many of these real options occur naturally (e.g., to defer, contract, shut down or abandon), while others may be planned and built-in at some extra cost (e.g., to expand capacity or built growth options, to default when investment is staged sequentially, or to switch between alternative inputs or outputs) (Trigeorgis, 1993).

The option to defer investment is analogous to an American call option on the gross present value of the completed project's expected operating cash flows, $V$, with the exercise price of the required outlay, $I$. Thus, it's value will be $\max(V - I, 0)$.

The option to default during construction (or the time-to-build option) can be valued similar to compound options approach of Geske (1979). The actual staging of capital investment as a series of outlays over time creates valuable options to "default" at any given stage. Thus, each stage is an option on the value of subsequent stages with exercise price of the installment cost outlay required to proceed to the next stage.

The option to expand is a call option to acquire an additional part (x%) of the base scale project, with a value of $\max(xV - IE, 0)$, where x is percentage rate of expansion and IE is the expansion outlay.
The option to contract is a put option and reflects management's flexibility to operate below capacity or even reduce the scale of operations, thereby saving part of the planned investment outlays. The value of this option is $\max(Ic - cV, 0)$, where $c$ is the percentage rate of reduction in the scale of operations and $IC$ is the associated cost saving.

The option to shut down (and restart) operations is a call option to acquire a particular year's cash revenues ($C$) by paying the variable operating cost ($Iv$) as exercise price, i.e., $\max(C - Iv, 0)$.

Options to alter the operating scale (i.e., expand, contract, or shut down) are typically found in natural resource industries, such as oil and mine operations, facilities planning and construction in cyclical industries, fashion apparel, consumer goods, and commercial real estate.

The option to abandon for salvage value is an American put option on current project value ($V$) with exercise price the salvage or best alternative use value ($A$), i.e., $\max(A - V, 0)$. Valuable abandonment options are generally found in capital intensive industries, such as airlines and railroads, in financial services, as well as in new product introductions in uncertain markets.

The option to switch use (i.e., inputs or outputs) is a valuable built-in flexibility to switch from the current input to the cheapest future input, or from the current output to the most profitable future product mix, as the relative prices of the inputs or outputs fluctuate over time. Input flexibility is valuable in feedstock-dependent facilities, such as oil, electric power, chemicals, and crop switching. Output flexibility is more valuable in industries such as automobile, consumer electronics, toys or pharmaceuticals, where product differentiation and diversity are important and/or product demand is volatile.

Corporate growth options: Many early investments such as research and development, a lease on undeveloped land or a tract with a potential oil reserves, a strategic acquisition, or an information technology network are prerequisites or links in a chain of interrelated projects. The value of these projects may derive not so much from their expected directly measurable cash flows, but rather from unlocking future growth opportunities. An opportunity to invest in a first-generation high tech product, for example, is analogous to an option on options (an interproject compound option). Growth options are found in all infrastructure based or strategic industries, especially in high-tech, R&D, or industries with multiple product generations or applications (e.g., semiconductors, computers, pharmaceuticals), in multinational operations, and strategic acquisitions.

Valuing Real Options: An Example

Consider an oil extraction project with extraction cost 208 million. Today's oil price is 40 dollar per barrel and the oil field has a capacity of 5 million barrel per year for two years. We expect oil price to go either up to 21 (80 percent) or down to 24 (40 percent). Risk free rate is 8 percent.
The NPV of this project is $(200 - 208 = -8 < 0$, and DCF analysis results in the rejection of the project. DCF analysis, however, ignores at least one option embedded in this project, the option to abandon the oil extraction at any time in exchange for its salvage value or value in its best alternative use if oil prices suffer a substantial decline. Assume that its best alternative use is land development. Land development has a value of 180 million which is below the project's value in its present use—otherwise management would have to abandon the project immediately. We believe that the value of the land development will go up 60 percent or down 20 percent.

![Binomial tree for oil prices and project value.](image)

**Figure 1:** Binomial trees for oil prices and project value.

This abandonment value is an American put option on current project value $(V)$ with exercise price the land development value $(A)$. This option entitles management to receive additional cash flow of $\max(A - V, 0)$.

![Binomial tree for the value of land development.](image)

**Figure 2:** Binomial tree for the value of land development.
Value of this option by using risk-neutral binomial option pricing formula\(^3\) is 13.33. Thus, the NPV of the project with option will be \((13.33 - 8 =) 5.33\) million.

This example supports McDonald and Siegel (1986)'s assertion that the simple net present value rule which is to invest as long as \(V > 1\) is incorrect. This rule is incorrect because it ignores the opportunity cost of making a commitment now, and thereby giving up the option of waiting for new information.

III. REAL OPTIONS AND INTERACTIONS BETWEEN FINANCING AND INVESTMENT DECISIONS.

Myers (1977) is the first to explain the relation between financing and investment decisions in a contingent claim framework. He describes the firm's potential investment opportunities as call options whose value depend on the likelihood that management will exercise them. If the firm has risky debt outstanding, situations arise in which exercising the option to undertake a positive net present value project potentially reduces share value because debtholders have a senior claim on the project's cash flows. Unless this conflict between the shareholders and debtholders is controlled, the probability that these real investment options will be exercised is reduced, thereby reducing firm value. One way to control this underinvestment problem and its associated value loss is to finance growth options with equity rather than debt (Smith and Watts, 1992). Hence Myers predicts that the larger the proportion of firm value represented by growth options (i.e., the lower the assets in place), the lower the firm's leverage, and the higher its equity-to-value ratio.

Hite (1977) presents a model in which he combines seemingly distinct theory of production and output, the theory investment and the theory of financial policy toward an integrated theory of the firm. His model is based upon discrete-time, continuous space variables. In other words he uses single-period CAPM with continuous demand and production functions. His model is static and does not consider the real options that a firm might have. The model presented here extends his model by using continuous-time, continuous space variables (i.e., continuous-time capital asset pricing model of Merton (1973) or Breeden (1979)). In addition our model incorporates various real options to reflect managerial flexibility in production, investment, and financing decisions.

\(^3\)C = \([pC^u + (1-p)C^d] / (1 + r)\) where \(C\) is call option price; \(p\) is risk-neutral probability (.4); and \(u\) and \(d\) refers to up and down states, respectively \((C^u = 0, C^d = 24\), \(r\) is risk-free rate (.08).

The risk-neutral probability is calculated by using: \(p = [(1 + r) P - p^d] / (p^u - p^d)\). \(P\) is price of the oil \((P = 40, p^u = 72, p^d = 24, \text{and } p = 0.4)\).
The Model:

Assume that the project value (cash flow) evolves according to the following geometric brownian motion:\(^4\)

\[
dV = a \, V \, dt + s \, V \, dz
\]  
(1)

where \(dz\) is increment of a Wiener process, \(a\) is the drift rate, and \(s\) is instantaneous volatility per unit of time. Equation (1) implies that the current value of the project is known, but future values are lognormally distributed with a variance that grows linearly with the time horizon. Thus although information arrives over time (the firm observes \(V\) changing), the future value of the project is always uncertain.

Our goal is to maximize the value of the investment opportunity, \(F(V)\) (that is, the value of the option to invest):

\[
F(V) = \max E \left[(V_T - I) \, e^{-qT} \right]
\]  
(2)

where \(E\) denotes the expectation, \(T\) is the (unknown) future time that the investment is made, \(q\) is a discount rate, and the maximization is subject to equation (1) for \(V\).

Assume that stochastic changes in \(V\) are spanned by the existing assets in the economy. This assumption will let us to make use of contingent claim analysis.

Let \(x\) be the price of an asset or dynamic portfolio of assets perfectly correlated with \(V\). Since \(x\) is perfectly correlated with \(V\), \(\text{corr} (x, M) = \text{corr} (V, M)\) where \(M\) is the market portfolio. We will assume that this asset or portfolio pays no dividends, so its entire return is from capital gains. Then \(x\) evolves according to

\[
dx = m \, x \, dt + s \, x \, dz
\]  
(3)

where \(m\), the drift rate is the expected rate of return from holding this asset or portfolio of assets. According to CAPM, \(m\) should reflect the asset's systematic (nondiversifiable) risk.\(^5\) \(m\) will be given by

\[m = r + \phi \, \text{corr} (x, M) \, s\]

---

\(^4\)This is a simplistic assumption. More realistically, one might argue that project value follow some different stochastic process. For example, one might believe that over long periods of time, oil prices (and the prices of other commodities) are drawn back towards long-run marginal cost, and thus are mean reverting.

\(^5\)Many real options involve underlying assets that have no systematic risk. For example, the risk of finding oil in an exploratory well is unsystematic, as is the research and development risk. Moreover, when the underlying asset has no systematic risk, the risk neutral probability is the same as the true probability (Sick, 1990).
where $r$ is the risk-free interest rate, and $\mathcal{O}$ is the market price of the risk.\footnote{That is, $\mathcal{O} = (r_M - r) / s_M$, where $r_M$ is the expected return on the market, and $s_M$ is the standard deviation of market return.} Thus $m$ is the risk adjusted expected rate of return that investors would require if they are to own the project.

Now, consider the following portfolio: Hold the option to invest, which is worth $F(V)$, and go short $n = F^*(V)$ units of the project (or, equivalently, of the asset or portfolio $x$ that is perfectly correlated with $V$). The value of this portfolio is

$$Q = F - F^*(V) V$$  \hspace{2cm} (4)$$

and its value is obtained by solving the fundamental equation of asset valuation:

$$1/2 s^2 V^2 F''(V) + (r - k) V F'(V) - r F = 0$$  \hspace{2cm} (5)$$

where, $k = m - a$ and the boundary conditions are:

$$F(0) = 0$$  \hspace{2cm} (6)$$

$$F(V^*) = V^* - I$$  \hspace{2cm} (7)$$

$$F'(V^*) = 1$$  \hspace{2cm} (8)$$

Condition (6) arises from the observation that if $V$ goes to zero, it will stay at zero (i.e., bankruptcy). $V^*$ is the price at which it is optimal to invest. Equation (7) just says that upon investing, the firm receives a net payoff $V^* - I$. The condition (8) is the smooth-pasting condition.

An example of this model is oil extraction project where $V$ is the value of this project and $x$ is the oil price per barrel. We can apply this approach to Hite's model by defining $V$ as firm value and $x$ as net (net of operating costs) cash flows.

So far we have explained how to apply contingent claim analysis to valuation of the firm. However, our main goal is to incorporate real options to this valuation, since it substantially affects the value of the firm and the risk adjusted required rate of return. Assume that operation of the firm will temporarily and costlessly suspended when cash flows $(x)$ falls below a flow cost $(c)$. Therefore at any instant the net cash flow from this project is given by

$$y(x) = \max(x - c, 0)$$

With this real option firm value satisfies the differential equation in (5) plus $y(x)$.

The solution to this model can be obtained by using stochastic calculus. With the inclusion of multiple real and financial options, however, it is not an easy task if not possible. In such an attempt Mauer and Triantis (1994) uses numerical solution techniques:
Mauer and Triantis analyze the interaction between investment and financing decisions in a multiperiod contingent claims model where the firm has flexibility to dynamically manage both decisions over time. They find that production flexibility has a positive effect on the value of interest tax shields. The ability to shut down operations allow the firm to mitigate operating losses. Therefore, as operating adjustment costs decrease, firm value increases and firm value variance decreases, increasing the debt capacity of the firm and the associated net benefit of interest tax shields. However, production flexibility and financial flexibility are to some degree substitutes, since the effect of lower operating adjustment costs on net tax shield value is less pronounced the smaller are recapitalization costs. They also examine the effect of production flexibility on the firm’s optimal dynamic recapitalization policy. As operating adjustment costs decrease, the average leverage ratio increases and the range over which the firm allows its optimal leverage ratio to vary without recapitalizing decreases.

In contrast, they find that debt financing has a negligible impact on the firm’s investment and operating policies. This is also a contradiction to Hite’s first proposition that the financing policy cannot be ignored in choosing the optimal productive technique. For example, while a levered firm has an incentive to invest earlier (i.e., at a lower commodity price) than an equivalent unlevered firm because it earns interest tax shields when it is operating, the benefit from doing so is largely offset by a loss in the value of waiting to invest. Therefore, the net benefit is not large enough to effect a significant change in investment policy. Similarly, their analysis indicates that any additional interest tax shields that a levered firm can earn by deviating from the optimal operating policy of an equivalent unlevered firm are counterbalanced by a loss in the value of its operating options. Thus a levered firm has little incentive to alter operating policy. From a practical standpoint, the implication is that the firm can determine exercise timing decisions on its real options, ignoring the effect of debt financing. However, since their results are numerical rather than analytic, they depend on the choice of parameter inputs.

IV CONCLUSION

This paper investigates the role of real options in capital budgeting, investment, and financing decisions. The net present value or other discounted cash flow approaches to capital budgeting fail to reflect real option values in capital budgeting decisions and therefore may lead to wrong decisions in acceptance or rejection of the project. Real options in the form of production flexibility has a significant effect on financing decisions. In contrast, financial policy has a minimal effect on the firm’s initial investment decision and subsequent operating. However these results are sensitive to choice variables.

Real options approach is rich in real-life applications and fruitful in future research. Extending it, for example, by using Bayesian analysis or alternative (e.g., jump) processes is only one among many directions for research.

7In general, a firm that employs at least some leverage will not employ the same capital-labor ratio that would be optimal if the firm were unlevered.
REFERENCES


Kogut, Bruce, 1991, Joint ventures and the option to expand and acquire, Management Science 37 (1), 19-33.


McDonald, Robert and Daniel R. Siegel, 1985, Investment and the valuation of firms when there is an option to shut down, International Economic Review 26 (2), 331-349.


Myers, Stewart C., and Saman Majd, 1985, Calculating abandonment value using option-pricing theory, Sloan school working paper # 1462-83, M. I. T.


